

Polynomial Long Division

Old School Long Division: (Yes that stuff from elementary school...)

Examples:

$$\begin{array}{r} 107 \\ 25 \overline{) 2675} \\ \underline{-25} \\ 175 \\ \underline{175} \\ 0 \end{array}$$

$$\begin{array}{r} 82 \\ 12 \overline{) 993} \\ \underline{-96} \\ 33 \\ \underline{-24} \\ 9 \end{array} \leftarrow \text{remainder}$$

$$82 \frac{9}{12} \text{ or } 82 \frac{3}{4}$$

You try:

$$\begin{array}{r} 5826 \\ 10 \overline{) 58260} \\ \underline{50} \\ 82 \\ \underline{-80} \\ 26 \\ \underline{-20} \\ 60 \end{array}$$

$$5,826$$

$$\begin{array}{r} 464 \\ 7 \overline{) 3248} \\ \underline{-28} \\ 44 \\ \underline{-42} \\ 28 \end{array}$$

$$464$$

$$\begin{array}{r} 1347 \frac{4}{9} \\ 9 \overline{) 12,127} \\ \underline{-9} \\ 31 \\ \underline{-27} \\ 42 \\ \underline{-36} \\ 67 \\ \underline{-63} \\ 4 \end{array}$$

$$\begin{array}{r} 4802 \\ 12 \overline{) 57628} \\ \underline{-48} \\ 96 \\ \underline{-96} \\ 02 \\ \underline{-0} \\ 28 \\ \underline{-24} \\ 4 \end{array}$$

$$4,802 \frac{4}{12} \text{ or } 4,802 \frac{1}{3}$$

Polynomial Long Division: You can do the same long division process with polynomials!

Example 1:

$$\frac{27x^3 + 9x^2 - 3x - 10}{3x - 2} = 9x^2 + 9x + 5$$

We would set this problem up as

$$\begin{array}{r} 9x^2 + 9x + 5 \\ 3x - 2 \overline{) 27x^3 + 9x^2 - 3x - 10} \\ \underline{-27x^3 + 18x^2} \\ 27x^2 - 3x - 10 \\ \underline{-27x^2 + 18x} \\ 15x - 10 \\ \underline{-15x + 10} \\ 0 \end{array}$$

$$9x^2(3x - 2) - (27x^3 - 18x^2) \Rightarrow -27x^3 + 18x^2$$

$$9x(3x - 2) - (27x^2 - 18x) \Rightarrow -27x^2 + 18x$$

$$5(3x - 2) - (15x - 10) \Rightarrow -15x + 10$$

Example 2:

$$\frac{x^4 + x^3 + 7x^2 - 6x + 8}{x^2 + 2x + 8} = x^2 - x + 1$$

$$\begin{array}{r} x^2 - x + 1 \\ x^2 + 2x + 8 \overline{) x^4 + x^3 + 7x^2 - 6x + 8} \\ \underline{-x^4 - 2x^3 - 8x^2} \\ -x^3 - x^2 - 6x + 8 \\ \underline{+x^3 + 2x^2 + 8x} \\ x^2 + 2x + 8 \\ \underline{-x^2 - 2x - 8} \\ 0 \end{array}$$

$$\begin{aligned} & x^2(x^2 + 2x + 8) \\ & -(x^4 + 2x^3 + 8x^2) \Rightarrow -x^4 - 2x^3 - 8x^2 \\ & -x(x^2 + 2x + 8) \\ & -(-x^3 - 2x^2 - 8x) \Rightarrow x^3 + 2x^2 + 8x \\ & 1(x^2 + 2x + 8) \\ & -(x^2 + 2x + 8) \Rightarrow -x^2 - 2x - 8 \end{aligned}$$

Example 3:

$$\frac{27x^3 + 9x^2 - 3x - 9}{3x - 2} = 9x^2 + 9x + 5 + \frac{1}{3x - 2}$$

$$\begin{array}{r} 9x^2 + 9x + 5 \\ 3x - 2 \overline{) 27x^3 + 9x^2 - 3x - 9} \\ \underline{-27x^3 + 18x^2} \\ 27x^2 - 3x - 9 \\ \underline{-27x^2 + 18x} \\ 15x - 9 \\ \underline{-15x + 10} \\ 1 \end{array}$$

$$\begin{aligned} & 9x^2(3x - 2) \\ & -(27x^3 - 18x^2) \Rightarrow -27x^3 + 18x^2 \\ & 9x(3x - 2) \\ & -(27x^2 - 18x) \Rightarrow -27x^2 + 18x \\ & 5(3x - 2) \\ & -(15x - 10) \Rightarrow -15x + 10 \end{aligned}$$

Use polynomial division to simplify each of the following quotients.

a) $\frac{x^4 + 3x^3 - x^2 - x + 6}{x + 3}$

b) $\frac{2x^4 - 5x^3 + 2x^2 + 5x - 10}{x - 2}$

c) $\frac{7x^4 - 10x^3 + 3x^2 + 3x - 3}{x - 1}$

d) $\frac{2x^4 + 8x^3 - 5x^2 - 4x + 2}{x^2 + 4x - 2}$

e) $\frac{3x^4 - x^3 + 8x^2 + 5x + 3}{x^2 - x + 3}$

f) $\frac{3x^4 + 9x^3 - 5x^2 - 6x + 2}{3x^2 - 2}$

g) $\frac{x^3 - 2x^2 - 4}{x - 2}$

h) $\frac{x^3 - 4x^2 + 9}{x - 3}$

i) $\frac{x^4 - 13x - 42}{x^2 - x - 6}$